INVESTIGATION OF STRESSES IN FACETTED GLASS SHELL STRUCTURES

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Summary

The typical use of triangular and quadrangular facets in doubly curved facetted shells requires the use of triangulated truss systems or quadrangular truss framing with diagonals or cross tension cabling. In such a structure, the load carrying ability is based on concentrated forces in the framing system, while the glass merely serves as a separation of the inside environment from the outside. In this paper facetted glass shell structures with three way vertices, i.e. with three adjoining edges in each vertex are considered, since the load carrying ability of such a structure is achieved primarily by in-plane forces in the facets and the transfer of distributed in-plane forces across the joints. It is described how these facets work structurally, specifically how bending moments develop and cause possible stress concentrations in the corners, which are subjected to uplift. Apart from local bending moments from distributed load, other types of bending moments are likely to occur, especially if the shell has areas of low stiffness, for example along a free edge. A facetted shell structure has been modelled in a finite element program, and the resulting stresses are presented and discussed.

Keywords: Shell structures, plane-based faceting, structural glass, structural duality, finite element analysis, in-plane action, bending action

1. Introduction

In order to introduce plane structural elements in a shell structure, without loosing the huge advantages of the doubly curved shell shape, facetted shell structures are considered. A facetted shell structure has a piecewise plane geometry, and together the facets form an approximation to a smoothly curved surface.

The process of faceting a smooth surface of double curvature can be divided into two principally different approaches: Point-based faceting and plane-based faceting [1]. These are described in section 2, “Structural behaviour of facetted shells”.

A distributed load on a plane-based facetted structure will cause bending moments in the loaded facets. Via this bending, the load is transferred to the facet edges, and if the rigidity of the structure is adequate, the load is subsequently carried via membrane forces in the shell [2]. This discrete transformation of the load into membrane forces is basically what separates a facetted shell structure from a smooth shell structure, where the transformation of the load into in-plane forces happens continuously. Also, a hinged joint line between two glass facets weakens the structures ability to transfer bending moments across the joint line, and this affects the direction and magnitude of the stresses in the shell.

The structural properties of glass are frequently described, and dealt with in different ways. The key challenge is of course the brittle nature of the material, which is troublesome when we wish to design a ductile and robust structure, while at the same time using glass in key load carrying elements. The structural behaviour of a facetted shell with a plane-based faceting system makes it ideal for the use of glass as the material. Some of the reasons are as follows:

- If shaped in the right way, the shell can carry considerable loading via low in-plane stresses, and thereby minimizing bending of the glass.

- The forces are distributed as in-plane stresses in the facets, hence avoiding the need of framing systems along the edges. Furthermore, load is transferred from one facet to another by distributed forces along the edges.

- The structure can be designed with a very high degree of transparency, since no additional bracings are needed.

2. Structural behaviour of facetted shells

As mentioned in the introduction, the faceting process can be divided into two principally different approaches: Point-based faceting and a plane-based faceting [1]. In a point-based system, three points define a plane. By scattering points on a smooth surface and connecting them with straight lines (while obeying certain rules), a triangulated geometry of piecewise plane triangles can be defined.

In a plane-based system, the smooth surface is approximated by a number of planes. In this system, three planes define a point. As a result, three – and only three – facets intersect in all the facet corners.

The structural behaviour of the two systems corresponding to the two different methods of faceting is very different from each other. In the point-based triangulated system the forces are concentrated forces in the framing system along the facet edges. This is a configuration we have seen in many built shell structures, where steel or wooden framing constitutes a triangulated convex load carrying system (or alternatively quadrangular with diagonal pre-stressed cable bracings), and the holes are clad with glass. In these cases, the glass is not active in the load carrying process.

In a plane-based facetted system the forces are distributed in the facets and are transferred between the facets mainly as in-plane membrane forces. Even though plane-based shell structures are a common sight among structures in nature, see e.g. [3], man made shells of this configuration are very rare, and they are far from as thoroughly investigated as the triangulated shell structures.

In the following, the structural behaviour of plane-based facetted shells is discussed. A possible faceting process inspired by a geodesic dome is discussed in [1]. This process has been used to generate the facetted shape which is investigated in [2] and in the present paper, see figure 1.

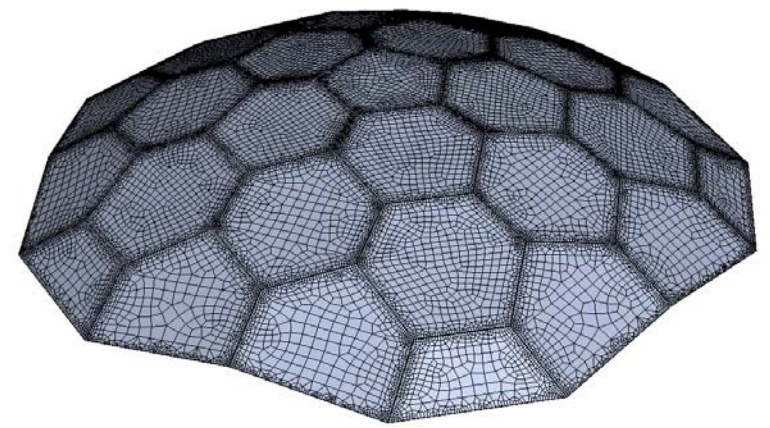

Fig.1 Finite element model of facetted shell structure.

Consider a facet in a plane-based facetted convex shell structure, loaded perpendicular to the facet surface. Local in-plane moments in the facet will distribute the load to the edges of the facet. The out-of-plane shear stresses, which thereby act at the facet edges, will be resolved into in-plane normal stresses in the facet itself and in the neighbouring elements, as illustrated in principle in figure 2.

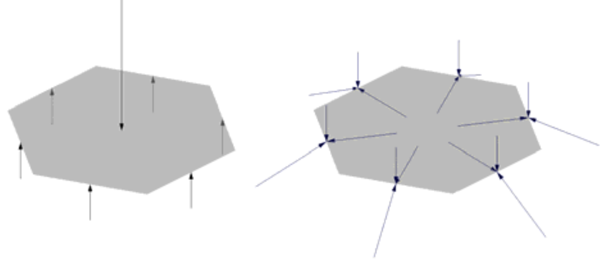

Fig.2 Principle sketch of local bending action in a facet, and resolving of the out-of-plane reaction forces into in-plane stresses at the facet edges.

However the load is not evenly distributed along the edges of the facet, mainly because the corners will be subjected to uplift. The local bending moments are described in greater detail in section 3, “Bending moments in the facets”.

In order to carry the load via membrane stresses, the shell structure has to be sufficiently supported. The best way to describe this is first to consider a triangulated shell structure, where all forces lie in the bars and nodes (edges and vertices). If this structure is not sufficiently supported, see e.g. [4] and [5], it is moveable (a mechanism). This can be either the whole structure or part of the structure. In the “structural space” (i.e. combinations of topology, geometry and support conditions) between a moveable structure and a rigid structure there may exists structures which are rigid in principle, but where the deformations are large and the forces in the bars may be considerable. This structure is naturally not suitable as a load carrying shell design. As described in [1], if a given triangulated shell structure is rigid in space with given support conditions, the dual (plane-based) structure will also be rigid as membrane structure under dual support conditions. A plane-based facetted shell structure (with three way vertices) which is not adequately supported – and thereby moveable as a membrane structure – is *not* necessarily a mechanism like the dual triangulated structure. It is able to transfer torsional moments between the facets and this may keep it from being moveable, though rather flexible. An example is shown in figure 3. The two structures are identical except from their support conditions. They are loaded by the same vertical distributed load. The structure to the left is supported in all three directions along its entire edge, and the structure to the right is only supported in three discrete points. There is a scale difference between the two plots. The deformations of the structure to the left are scaled up 4500 times more than the deformed structure to the right.

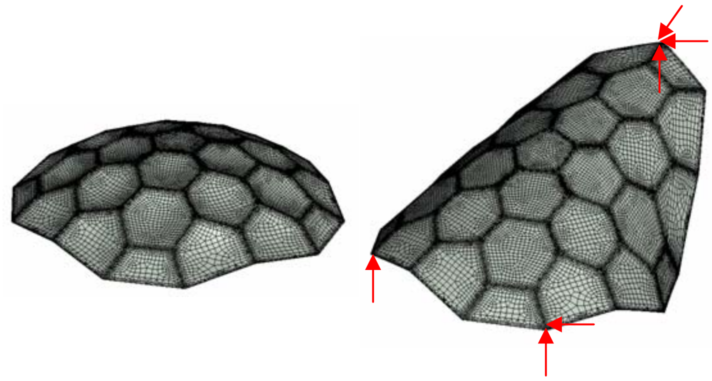

Fig.3 Deformed plot of two identical, but differently supported, facetted shell structures loaded by the same vertical load. The scale difference between the two plots is 4500.

Transferred torsional moments can also develop in an area of a plane-based structure, which is in fact rigid as a membrane, but has relatively large deformations. The transfer of torsional moments will be discussed in the next section.

3. Bending moments in the facets

Consider a rectangular plane plate with a constant thickness, simply supported along all four edges. If loaded by a uniformly distributed pressure load perpendicular to the plate surface, the plate will be subjected to bending moments. In the following, the directions of the edges (which in this case are orthogonal) are referred to with index x and y. Along the edges the bending moments m_{xx} and m_{yy} are zero, and only torsional moments m_{xy} can occur. The principal directions of the moments along the edges are thus angled 45 degrees with the edge. The torsional moments reach their maximum in the plate corners. Along the angle bisector in the corner m_{xy} can be interpreted as a hogging moment, resulting in the well known uplift of the corner. According to classical plate theory (Kirchhoff plate theory, see e.g. [6]), this uplift is balanced by a concentrated reaction force in the corner, in the same direction as the load. When the problem is described according to the more advanced Mindlin plate theory, which includes shear deformations in the plate, this corner force is distributed over a small length of the edges adjoining in the corner.

The directions of the principal moments are by definition orthogonal, and naturally this is also the case for the direction of the principal curvatures. In the simply supported rectangular plate, the edges are orthogonal due to the geometry and remain straight due to the support condition. The direction of the principal moments and the principal curvatures along the edges must therefore be oriented with an angle of 45 deg. with the edges.

The reaction forces vary in a parabolic fashion along the plate edges, from a value of zero at the corners to the maximum value at the middle of each edge. However, as described above there is a concentrated reaction force in the opposite direction in each corner, see [6].

Now consider a hexagonal plate with similar load and support conditions. All corner angles are between 90 and 180 deg. Due to the support conditions along the edges, the deformed plate has zero curvature in the direction of the edge and perpendicular to this. This gives us a geometric and structural contradiction in the corner, where the principal directions will not be orthogonal, since this is neither geometrically nor structurally possible. For an analytical description of the problem see [7]. This results in a singularity in the corner, with resulting indefinite bending stresses.

Finally, consider a hexagonal plate, like the one described above, but with a spring support against translation perpendicular to the surface along the edges. The plate edges are also supported by a continuous rotational spring opposing rotations around the edge direction. This modification of the support condition allows for the edge to slightly change shape, so that the direction of the principal curvature is not forced to be oriented 45 deg. to the edge. This removes the bending moment singularity in the corner, but depending on the stiffness of the springs, it can still lead to stress concentrations near the corners.

The plates in the facetted shell are, like the last plate described above, supported with translational spring supports and a rotational spring support against rotations around the edge direction. Both the translational and rotational springs are constituted by a joint with a definite stiffness, and of the flexibility of the adjacent plates. The shell structure in figure 1 has been calculated in the finite element program ABAQUS. The elastic modulus of the glue in the joint is 250 MPa and the width of the joint is 10 mm. The stress distribution shows only small stress concentrations in or near the corners.

As mentioned above, the considered plate is supported by the adjacent plates. The out-of-plane reaction forces along the plate edges are resolved into in-plane stresses in the adjacent plates and in the considered plate itself, as shown in the principal sketch in figure 2. The corner uplift tendency will cause concentrated forces at the corners. The vertex cannot raise freely, since the three plates are not in the same plane. The plates will have an in-plane stiffness against lifting, and this resistance can be seen from the plot of the membrane stresses in the vertex area, see figure 4. The plot to the left is a contour plot of the maximum in-plane stress around a vertex, and the plot to the right is a vector plot of the principle in-plane stresses in the same area.

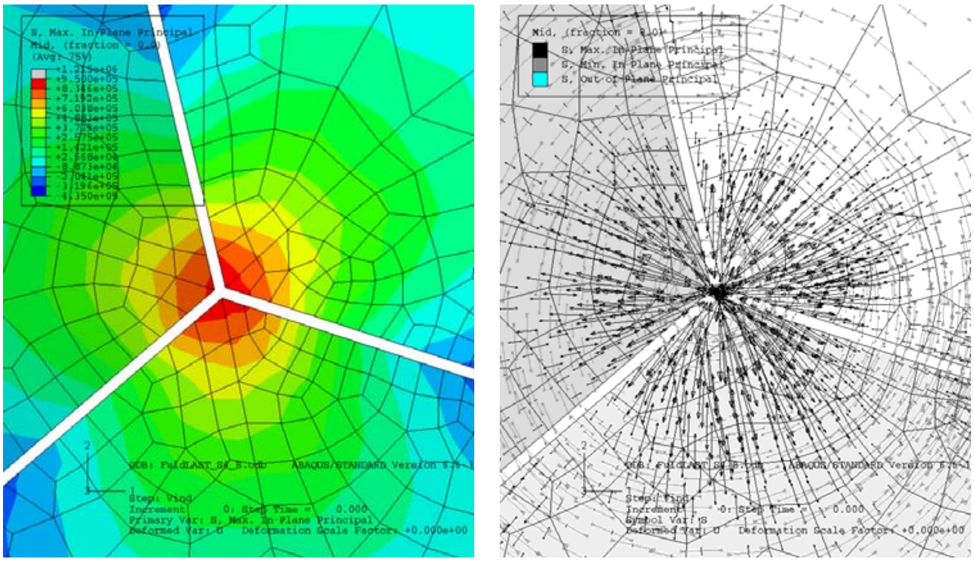

Fig.4 Membrane stresses in a vertex of the facetted shell, for a uniformly distributed vertical load of 0.72 kN/m². The glass plate thickness is 10 mm. The calculated structure is described in section 5.

If the connection is designed so that the plates cannot transfer loads between them in an area around the vertex, the corners will simply lift when the plate is loaded. In this case, the in-plane stresses from the in-plane resistance from the uplift of the plate corners are avoided. This relieves the joint from relatively high stresses and has little influence on the stiffness of the structure.

4. Transfer of bending moments between plates

As described in section 3 the plates are affected by local bending moments when loaded out of their own plane. The stiffness of the joint, with regards to the rotation around the edge direction, determines how large a part of this bending moment is transferred into the adjacent plate. If the rotational flexibility of the joint is infinitely stiff, the plate is continuously connected to the adjacent plate, and if both plates are equally loaded, the plate will approximately behave as if clamped along the edge (i.e. fully supported against rotation around the direction of the edge). If the joint has a rotational spring stiffness of zero, the plate will approximately be simply supported along the edge.

The stiffness of this rotational spring can be interpreted as a “degree of clamping”. If the rotational stiffness is zero the degree of clamping is 0%, and if the rotational stiffness is infinitely large, the clamping effect is 100%. The larger degree of clamping, the larger stresses will have to travel through the joint. Thus, it is possible to formulate a suitable relation between the stiffness of the joint and the strength of the joint.

If a given convex triangulated lattice structure is moveable as a mechanism, the dual plane-based facetted shell structure will not function as a membrane structure. However, because of the ability of the latter structure to transfer torsional moments between the facets, it will not behave as a moveable mechanism. Because it is able to carry load via bending as well as membrane action, a plane-based facetted shell structure will therefore never be statically determinate.

Consider a connection line in a plane-based facetted shell structure, and the two facets adjacent to this connection, as shown in figure 5. The direction of the joint is referred to as l and the direction perpendicular to this and lying in the plane of one of the facets is referred to as u. A direction perpendicular to l and lying in the plane of the other facet is referred to as v.

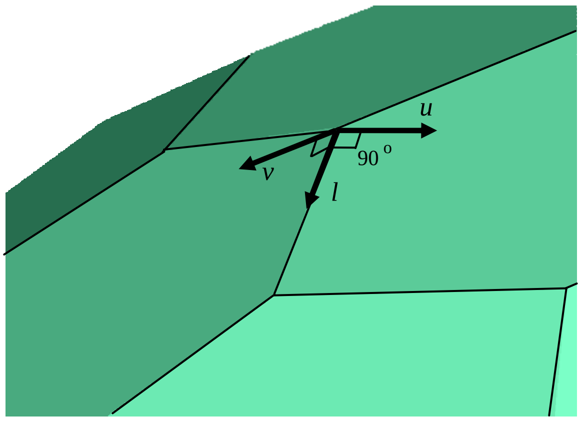

Fig.5 A connection line in a plane-based facetted shell structure.

The moment m_{uu} in the connection is minimized by the flexibility of the joint. At the same time the two plates stiffen each other along their joint line, since they are not lying in the same plane. This has the effect that the deformed joint line is almost straight, and therefore the moment m_{ll} in the two plates is small close to the joint line.

This leaves the torsional moment, m_{ul} in one plate and m_{vl} in the other as the predominant bending moment along the joint. The transfer or equilibrium of torsional moments between neighbouring plates have to be assessed through vector decomposition of the moment vectors, or most easily through the use of the equivalent transverse forces which can be decomposed into in-plane and out of plane forces. This means that transfer of torsional moments depends on the relation between out of plane stiffness and in-plane stiffness of the plates.

FE-calculations of two identical structures with different support conditions have been carried out. The in-plane principal stresses in a vertex are shown in figure 6. The structure plotted to the left is a rigid membrane structure, and here the torsional moment along the edge is transformed into in-plane action. The structure plotted to the right is insufficiently supported as a membrane structure, and therefore the torsional moment in one plate is transferred across the joint primarily as out of plane shear forces, which affects the neighbouring plate with approximately the same torsional moment.

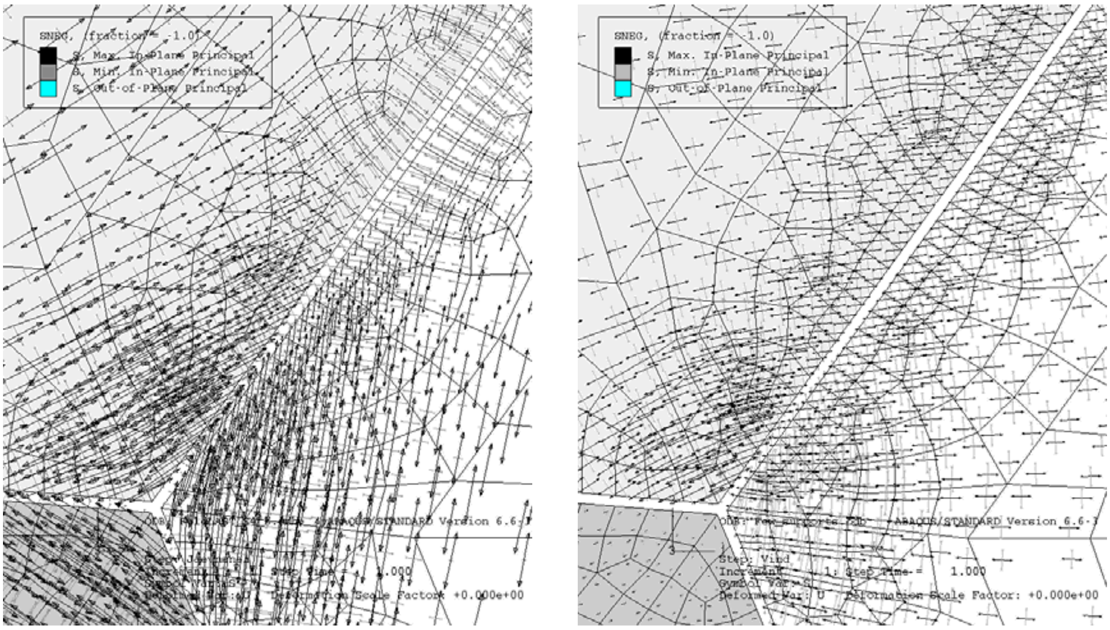

Fig.6 Principal stresses on the lower surface of a plane based facetted shell structure loaded by self weight. The image to the left is from a structure which is supported so that it is rigid as a membrane structure, and the image to the right is from a structure which is not sufficiently supported to function a membrane structure. The scaling of the vectors are different in the two plots.

5. Calculated example of facetted shell structure

An example of a facetted shell structure of glass has been calculated in the finite element program ABAQUS. Some of the results of this calculation are presented in the following.

The modelled structure is a convex plane-based facetted shell structure, where all facets are tangent planes to the same paraboloid of revolution, see figure 1. The maximum span is about 11 metres. The facets measure roughly 2 metres diagonally. The structure is supported against translation in all three directions along the boundary.

The load is composed of self weight (corresponding to 10 mm glass panes), snow load (0.72 kN/m^2 vertical load) and wind load (varying from a pressure force of 0.15 kN/m^2 perpendicular to the facets in one side to a suction at the top of 0.42 kN/m^2, and decreasing to a suction of 0.04 kN/m^2 at the opposite side). The facets are all 10 mm glass plates with a stiffness of 70 GPa and a specific gravity of 25 kN/m^3. The joint between the facets are modelled as glued butt joints, with a stiffness of 250 MPa and a width of 10 mm. The stiffness of the joint is not reduced in the corner areas.

The FE-model consists of thin shell elements, which do not include out of plane shear deformations. All calculations are linear, corresponding to small deformations. Convergence analysis has been carried out to confirm the calculated stress distribution, with particular focus on the glass plate corners. These analysis have shown that the calculated stresses in the glass facets has converged acceptably when given the mesh size shown in figure 1, but the stresses in the glued butt joint have not converged. The latter is due to the coarse modelling of the joint. An analysis of the joint has not been the main purpose of the investigation.

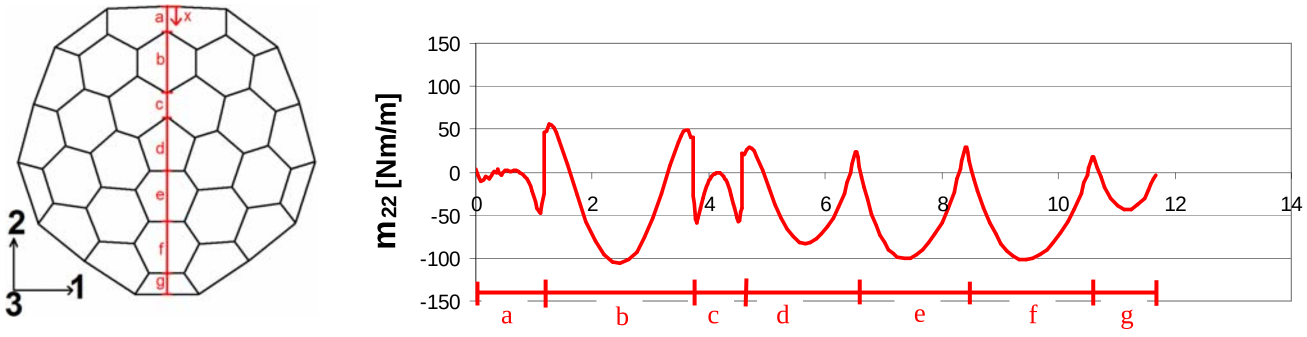

Fig. 7 Bending moment m_{22} along the indicated line across the structure for uniform snow load. A bending moment of 100Nm/m corresponds to a tension of 6MPa in the glass surface from bending.

Figure 7 shows the bending moment m_{22} for uniform snow load along the indicated line, in relation to the main axes shown in the figure. The main axes and the indicated line are arranged so that they are parallel to some connection lines and perpendicular to others. At section *a* and *c* the values are taken from the glass edge to the immediate right of the connection line.

A certain degree of clamping caused by the stiffness of the modelled joint can be seen e.g. in section *e*. The given joint stiffness results in a degree of clamping of about 15% relative to a full clamping.

From the bending moment in section *c* it is evident that the edge line does not remain a straight line. This is due to the uplift of the corners. In section *b* the hogging moment from the resistance to a free uplift of the corner is apparent.

For self weight and uniformly distributed snow load, almost no in-plane shear stresses develop between the facets. This is because the shape of the structure is close to the optimal shape given an evenly distributed load (which the snow load is, and the self weight almost is). Only in-plane normal forces develop between the facets. The in-plane normal stresses in the glass along the edge is 0.14 MPa (or 1.4 N/mm) for self weight and 0.43 MPa (or 4.3 N/mm) for the uniform (characteristic) snow load.

For non-uniform load in-plane shear forces develop between the facets. For wind load the largest in-plane stress between two facets is 0.30 MPa (or 3.0 N/mm). A non-uniform contribution to the snow load consisting of a variation between 0.72 kN/m^2 and –0.72kN/m^2 results in a maximum in-plane shear stress of 0.48 MPa (or 4.8 N/mm). The size of the in-plane normal forces for a non-uniform load varies with the load on the regarded facets.

6. Discussion

A shell structure of glass combines a highly effective structural principle with a material of optimal permeability to light. With a facetted geometry, the structural principle of the shell structure is combined with a plane element shape, which is therefore simple to describe, prefabricate and transport. If the faceting process is plane-based only a minimum of structural material is necessary in addition to the facets themselves, namely a joint which is able to transfer distributed in-plane forces.

Many methods of handling the faceting process can be proposed. As long as it is plane-based, the structural principle is basically the same. Distributed in-plane forces, combined with local bending in the facets.

Numerous future research areas are of interest in the field of facetted shell structures.

- The design of the connection between the facets, while taking load transfer, redundancy, water tightness and other issues into consideration.
- Design of the connections to the supporting structure.
- Design of free edges, and how to ensure sufficient stiffness.
- A comparison to smooth shells of double curvature, and thereby developing ways of estimating structural load bearing capacity of a given facetted shell structure.
- Buckling behaviour of the facetted shell structure.

7. References

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