Facetted Shell Structure of Glass

Anne Bagger, Ph.D. Student, aeb@byg.dtu.dk, BYG-DTU, Technical University of Denmark
Brovej, Building 118, 2800 Kgs. Lyngby, Denmark
Jeppe Jönsson, jej@byg.dtu.dk, professor, BYG-DTU
Henrik Almegaard, hal@byg.dtu.dk, assistant professor, BYG-DTU
Ture Wester, ture.wester@karch.dk, associate professor, Royal Danish Academy of Arts, School of Architecture

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Abstract

In shell structures, which are appropriately shaped and supported, bending stresses are minimized, and loading is transferred primarily via in plane stresses (membrane stresses). This allows for a better utilization of the capacity of the structural material, since stresses are distributed evenly over the thickness of the structure in stead of concentrated at the surfaces. The stiffness to weight ratio of a shell structure is remarkably good, since the absorption of loads is provided by the overall global shape of the structure, and not a local sectional area. Glass is already widely used for load carrying structural members like fins, beams and columns. The structural use of glass is troubled by a brittle behaviour of the material, and a limited capacity for carrying tension forces. However, these characteristics can be taken into account in the design process in various ways. If the glass is used as the load carrying material in a shell structure, bending can be avoided, and the stress level can be minimized to a remarkably low level. In order to avoid the high production cost of doubly curved glass, facetted glass shell structures are considered. The faceting of a given curved surface can be done in many ways, but if the procedure is subjected to specific constraints, certain advantageous characteristics can be achieved. A plane-based faceting, where all vertices have three adjoining facets, results in a structure which carries load via in-plane stresses distributed in the facets, and the distributed shear along the edges. This corresponds well to glass being the load carrying material, since stress concentrations are avoided.

Introduction

Architects and engineers often seem to be working as two separate units. This has resulted in a gap between the two professions – the art of architecture and the art of engineering are no longer simply different perspectives of the same matter. It seems that cooperation between the two areas of expertise is limited to making demands and working out compromises, and that the art of conceptual design is diminishing. Structural morphology is the study of the interaction between geometrical form and load carrying behaviour. As a research subject, it belongs in the gap between architecture and engineering. The present paper is a study of the structural morphology of facetted shells, and the application of glass in such a structure.
A shell structure with a smooth surface will, if the shape of the surface and the support conditions are suitable, take the load primarily via in-plane stresses. Stresses from bending moments are thus minimized, and this allows for an exceptionally good usage of the construction material. Typically, the thickness/span ratio is as little as 1 to 500 for a reinforced concrete shell. For a span of e.g. 30 meters, this corresponds to a shell thickness of merely 6 centimetres. A more traditional solution with beams results in a typical ratio between span and construction height of 20 to 1, corresponding to beams with a height of 1.5 meters in the afore mentioned example.

In order to introduce plane structural elements in a shell structure, without losing the huge advantages of the doubly curved shell shape, facetted shell structures are considered. A facetted shell structure has a piecewise plane geometry, and together the facets form an approximation to the smoothly curved surface. The ways of faceting can be divided into two principally different approaches: Point-based faceting and plane-based faceting.
In a point-based system, three points define a plane. By scattering points on a smooth surface and connecting them with straight lines (while obeying certain rules), a triangulated geometry of piecewise plane triangles appears.

In a plane-based system, the smooth surface is approximated by a number of planes, which all locally are parallel to a tangent plane of the surface (if not *the* tangent plane). In this system, three planes define a point. As a result, three – and only three – facets intersect in all the facet corners.
The structural behaviour of the two systems corresponding to the two different methods of faceting are very different from each other. In the point-based triangulated system the forces lie in the edges and in the corners. In the plane-based system the forces lie in the facets and are transferred as shear between the facets. The two systems can be described as a dual pair. [1]

The structural properties of glass are described on numerous occasions, and dealt with in different ways. The key challenge is of course the brittle nature of the material, which is troublesome when we wish to design a ductile and robust structure, while at the same time using glass in key load carrying elements. The structural behaviour of a facetted shell with a plane based faceting system makes it ideal for the use of glass as the material. Some of these reasons are as follows:
- If shaped in the right way, the shell can take up considerable loading via low in-plane stresses, and thereby minimizing bending of the glass.
- The forces are distributed as in-plane stresses in the facets, hence avoiding stress concentrations along the edges. Furthermore, load is transferred from one facet to another by distributed forces along the edges.
- The structure can be made almost 100% transparent, since no additional bracings are needed.

Faceting a Smooth Surface

The term "faceting" should be comprehended as the transformation of a smooth surface of double curvature into a number of plane surfaces which form an approximation to the original surface. Only surfaces with positive Gauss curvature (i.e. convex shapes) are considered in the following.

As explained in the following section "Structural Behaviour of Facetted Shells", we wish to choose a faceting which results in three adjoining edges at each corner, as this form gives the structure certain desired static characteristics.
Furthermore, we wish to keep the facets approximately the same size and as regular in shape as possible. This is a question of aesthetic preference, of course, but a good starting point, which can later be manipulated.

Figure 1. Geodesic Dome by Buckminster Fuller. USA Pavilion at Expo '67, Montreal.

Buckminster Fuller patented in the 1950ies a method of triangulating a sphere (figure 1). As shown in figure 2 the regular triangles of one of the Platonic polyhedra with triangular faces (the icosahedron, the octahedron or the tetrahedron) are subdivided into smaller triangles. Subsequently, the corners of these triangles are

projected out onto the enveloping sphere, and connected with straight lines, forming a triangulation of the sphere. There is no perfect way to perform the subdivision of the polyhedrons faces, as neither the angles nor the sides can be preserved without one distorting the other when projected to the sphere.
The resulting triangulated sphere will have almost solely six-way vertices (i.e. the corners will have six edges connecting them to surrounding corners).
A small number of vertices will have fewer edges connected. If the triangulation is based upon an icosahedron (20 regular triangles with 12 vertices, each formed by 5 triangles) 12 of the vertices will be five-way, corresponding to the number of vertices in the icosahedron and the number of triangles in each vertex respectively. Likewise, if the triangulation is based upon an octahedron (8 regular triangles with 6 vertices, each formed by 4 triangles) 6 of the vertices will be four-way.

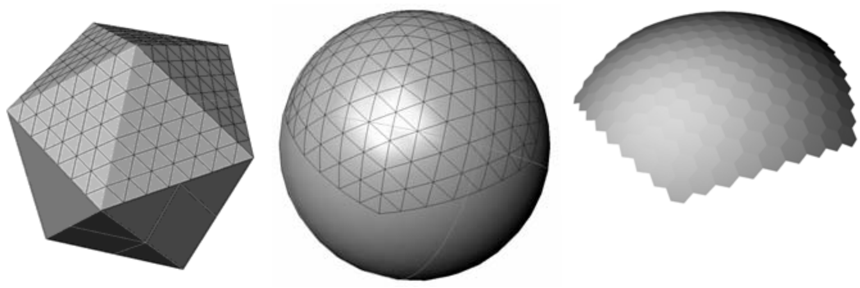

Figure 2. How to construct a Geodesic sphere from an icosahedron. The image to the right shows the plane based dual to the triangulated geometry in the centre image.

Almost the same procedure can result in a plane based faceting of the sphere which is not triangulated, but with solely three-way vertices. The procedure is followed until the point where all the corners of the triangles on the polyhedron are projected onto the surface of the enveloping sphere. Instead of connecting the corners, the tangent planes of the surface at the corner points are established, and constructed to intersect one another. The result of this is a geometry which is *dual* to the triangulated geometry. If a vertex in the triangulated system is six-way, the corresponding plane facet in the dual geometry will have six edges. Any given facetted geometry can be transformed into its dual. In this transformation, planes with *n* sides will become *n*-way vertices, and *n*-way vertices will become planes with *n* sides. This means that any triangulation of a surface can be transformed into a dual faceted geometry, where all vertices are three-way. Duality [1] will be discussed further in the section “Structural Behaviour of Facetted Shells”.

Structural Behaviour of Facetted Shells

Static duality
As described in the above section (“Faceting a Smooth Surface”), the geometric dual to a triangulated convex surface is a convex facetted geometry with three-way vertices. An example is shown in figure 3. Consider a triangulated convex geometry, adequately supported, so that it is rigid and fixed in space. A triangular plate has three lines of support, which is just enough to support the plate itself. If the plate is not externally loaded, these three shear forces will be zero. This can also be explained in the following way: If the triangular plates are removed, leaving only bars connecting the vertices, all the openings will be of triangular form, and therefore stable. Hence, the spatial stability and static behaviour of the structure does not depend on whether the holes are filled out or not, and therefore the triangular plates must be free of force unless directly loaded.
In the dual structure [1] to the one described above all vertices are three-way, and a similar but dual view can be taken: Every vertex has three lines of support, which is just enough to support the point itself. If the vertex is not externally loaded, these three normal forces will be zero. Explained in another way, if the vertices were to be cut off the structure with a cutting plane, the shape of the hole edges would be triangular, and thus the hole would be stable. Thus, the static behaviour of the structure does not depend on the vertices, and there are therefore no normal forces in the edges unless directly loaded. The spatial stability of the structure relies on in-plane forces in the facets, and shear transference between the facets.

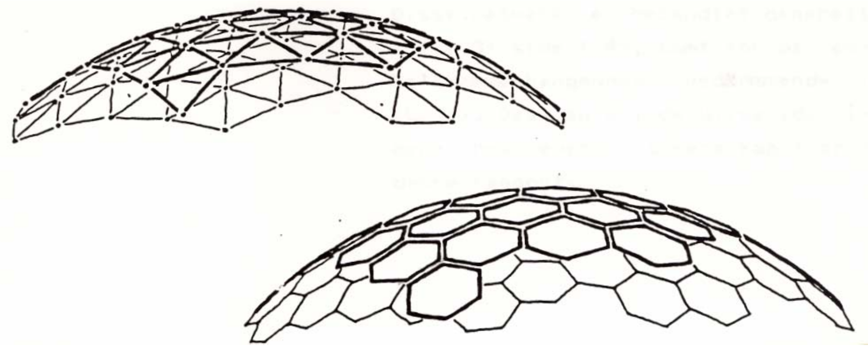

Figure 3. Two dual shells structures, with triangular facets and three-way vertices respectively.

Load bearing behaviour
Consider a convex structure, which is facetted so that all vertices are three-way. A load perpendicular to one of the facets will be transferred to the edges of the facet via bending moments within the facet. It is assumed that the joint is a perfect hinge. At the edge of the facet the load is decomposed into in-plane normal stresses in the element and in the neighbouring elements, as illustrated in figure 4. To the extend that these normal stresses from the loading of the facets are not in equilibrium, shear stresses between the facets will arise.

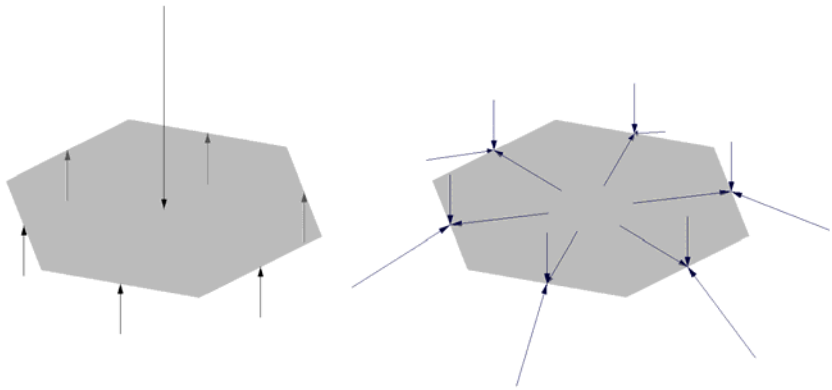

Figure 4. Local bending action in a facet, and decomposing of out-of-plane forces into in-plane stresses at the facet edges.

If a given triangulated shell structure is stable in space with given support conditions, the dual structure will also be stable as membrane structure under the same (but in principle dual) support conditions. Furthermore, it is likely that two structures which are each others dual will behave with a relatively similar stiffness against a given load. Further investigations must show if this can be exploited in the search of a sound load carrying plane based systems (with three-way vertices). With the software available at this point, lattice structures are much easier to describe and calculate than plate structures. Therefore, for a given plate structure, the dual triangulated structure could be evaluated, thereby giving a quick and reliable overview of the structural performance of the dual plate structure. If such a method can be developed, it will be extremely useful in a phase of conceptual design where many different solutions are investigated.
If all vertices in a triangulated convex shell have 6 lines/bars connecting them to the surrounding vertices, a useful method for determining the spatial stability already exist. *The Stringer System* [2] is a method of analysing membrane shells and designing their boundary conditions via a discrete truss model subjected to specific constraints. The Stringer System can naturally be applied to a system that is already discrete, and if the triangulation follows the geometric constraints of the Stringer System, i.e. that all vertices are six-way, the Stringer System is directly applicable for a qualitative overview of the spatial stability of the triangulated shell. However, the Stringer System cannot be used directly if vertices of lower valence that six are incorporated in the facetted geometry. Six-way vertices can constitute a regular geometric system in a plane, but will inevitably distort when forming a surface with a curvature. Therefore, it will often be of aesthetic interest to incorporate vertices of lower valence, thereby excluding the possibility of using the Stringer System directly.

An investigation of the spatial stability can be carried out with the Neutral Method [3], which reveals the *possibility* for spatial stability (as neutrality is only a necessary condition for stability). The Neutral Method compares the number of elements (nodes and/or plates, depending on the topology) to the necessary number of connectors (bars and/or shear lines), by assigning every element with a “stability number”.

Bending moments in facetted shells
A convex facetted shell with three-way vertices can be adequately supported, so that it can *just* carry the loads via membrane stresses. “Just” in this case means that fewer supports would result in the shell not being stable as a membrane. But even if this “just adequate support condition” is satisfied, the shell structure is not statically determinate, because the shell is able to transfer torsional moments as well as membrane stresses between the facets. However, the deformations corresponding to the membrane effect are a lot smaller than those corresponding to the bending effect, and therefore the effective in-plane stresses will develop much before bending occurs. If larger deformations can take place (e.g. if the structure has a relatively low stiffness) or if the deformed figure is somehow forced to go through considerable shape changes in certain areas of the structure (e.g. near inappropriately constraining support conditions) torsional bending moments will occur. The distribution and magnitude of these are a subject of further investigations.

Glass as load carrying material

Why glass is an obvious choice for a facetted shell structure
Every time glass is considered as a possible load carrying material in a given structural part, the question of how to deal with the brittle nature of the material arises. Structural ductility must be integrated into the structure by various means, not depending directly on the structural capabilities of the glass itself. The following issues must be dealt with:

- The load must be applied carefully to the glass, with no points of stress singularities and as low stress concentrations as possible.
- Broken structural elements must not pose a threat to the surroundings.
- A local failure of a part of the structure should expose itself with a visible warning, before a possible collapse of the entire structure occurs.

Figure 5. Glass dome with three-way vertices and glued joints. Designed by Mogens Jørgensen and Ture Wester.

Apart from being a structure of the utmost possible transparency (see figures 5 and 6), a facetted shell structure with three-way vertices is an exceptional conceptual design for a glass structure, since it by nature complies with the design issues described above.

- The main load carrying effect is constituted by relatively low in-plane stresses, distributed over the entire area of the facets.

- Loads are transferred into the glass via the full length of the edges, thereby minimizing stress concentrations in the connections between the structural parts.
- Laminating the glass-facets keeps broken glass pieces from falling down, and the structure from being too severely weakened if breakage of a single pane occurs.
- The shear transferring joints between the glass pieces can be designed so that holes in the glass are not necessary, thereby avoiding the stress concentrations otherwise caused by the holes.
- Since no normal forces run in the edges, no additional bracing system is needed and the only structural part besides the glass facets are joints transferring relatively low in-plane stresses. It is expected that such a joint can be designed with a width of 10 to 40 mm.

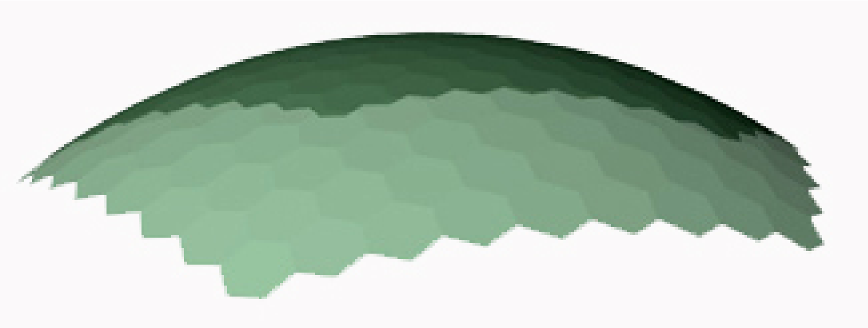

Figure 6. Rendering of glass dome with three-way vertices.

Moment singularities

Consider a simple plane plate with a given polygonal shape, loaded by a distributed load perpendicular to its plane. The plate material is linear elastic, and the moment distribution is calculated with either Kirchhoff or Mindlin plate theory. Moment singularities can arise in the corners, depending on the corner angle and the support condition along the edges adjacent to the corner [4]. The circumstances which lead to moment singularities are summarized in the following:

A corner with a 90 deg. angle does *not* result in a singularity, regardless of the support condition. If the angle is more than 90 deg. and less than 180 deg. (i.e. still convex), a simple support of the edges *will* lead to a singularity, and clamped or free edges will *not* lead to singularity. Corners with angles of more than 180 deg. (i.e. concave corners) *will* have a singularity regardless of the support condition.

For the hexagonal and pentagonal glass facets – which primarily will have corner angles lying between 90 and 180 deg. – this means that, depending on the design of the connection, moment singularities are a possibility which must be considered.

The glass facets differ in a number of ways from the above mentioned ideal plates. Some of these differences reduce the mathematically infinite stress singularities to definite stresses. Firstly, the corners of the facets are not perfectly sharp. Secondly, the corners are not fully supported against translation perpendicular the plate, as the plate is supported on other plates, which also have a definite stiffness against movement of the corner – and in most cases the surrounding plates are subjected to a similar load and thereby have a similar tendency to an out-of-plane translation of the corners. At the same time, no matter how the supports are constructed they are somewhere in between simple and clamped, and probably not fully homogeneous along the entire length of the edge. All these aspects result in a stress distribution different from the ideally calculated plates described above.

FE-calculations have been carried out, investigating the possible stress concentrations near the plate corners. The modelled structure is the facetted shell structure shown in figure 6. The structure is supported against translation in all three directions along the edge. The elements are thin shell elements assigned with the linear elastic isotropic properties of glass. The connected edges are 10mm wide strips of thin shell elements with a stiffness of 500MPa, simulating a glued butt joint. The structure is subjected to an evenly distributed vertical load.

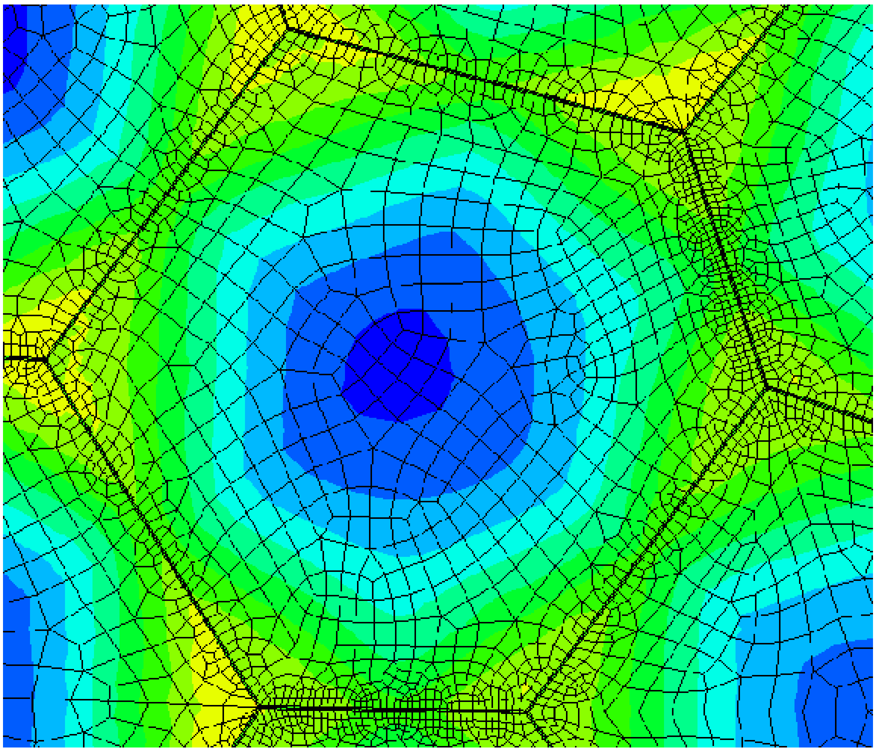

Figure 7. Result of the described FE-calculation. Largest in-plane principle stress in the upper side of the shell elements. The definite value of the stresses is not shown. The stress distribution does not imply moment singularities near the plate corners.

The FE-calculations does not imply stress singularities in or near the plate corners, as illustrated in figure 7. A convergence analysis of the stresses (through a refinement of the mesh) shows only small deviations of the stress values, and this consistency excludes the existence of singularities. Thus, the moment singularities have been softened by the geometric and static deviation from the above mentioned ideal plates, and stress concentrations are therefore not a design problem in this case. Further investigations must show whether singularities occur if other joint types are used, or if the angles between the facets are larger (as in the case of a more coarse faceting).

Discussion

The use of glass as the primary load carrying material in a facetted shell structure is a fascinating thought. If the design challenges can be solved satisfactorily, we are dealing with a structure which is almost 100% transparent, and which has very little material consumption, even at relatively large spans. The elements are plane, which makes them simple to describe, prefabricate and transport. The facetted geometry can be chosen with great artistic freedom, as long as the faceting is plane based. The stresses are quite low and well distributed over the surface, and it is possible to avoid stress concentrations if joints and supports are designed carefully. These characteristics make glass a possible choice for the load carrying material.

In the present paper the Geodesic Dome is suggested as the basis of the faceting procedure, resulting in a faceting where the facets are roughly of the same size. Other faceting procedures will result in other geometries. The key concern for the facets, for the glass panes to be active as load carrying elements, is that the faceting must be plane based, i.e. with three adjoining edges in each corner.

The field of facetted structures involves numerous interesting research subjects, such as the influence of the joint stiffness on stress singularities and clamping effects. The subject of torsional moments in the shell is to be investigated, and the transfer of these forces between the facets. Research is also needed in the practical design of the joint, and in the vast area of buckling analysis.

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