

THE STABILITY OF A FACETTED GLASS DOME STRUCTURE

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ABSTRACT

Since glass is a very strong material in compression, it seems the perfect material to construct shells structures of positive curvature with. It would result in a structural system based on compressive membrane stresses combined a very efficient materials in compression. However, to reach a high degree of efficiency a shell structure has to be doubly curved, and doubly curved glass elements are very expensive to produce. When considering dome-type structures based on membrane action in plane elements, Ture Wester [1] thought of the idea to combine the very efficient plate based dome structures with the structural use of glass.

In a Ph.D.-research at the Technical University of Denmark [2] this idea is being developed further by Anne Bagger. In this research a number of different problem areas have been acknowledged and one of them is the buckling behaviour of a faceted glass dome structure. Because of the compressive capacity of glass and the efficiency of the structural system, it is possible to create a slender structure. However, such a slender structure will also be highly susceptible to buckling, which is why a buckling analysis is likely to be critical. This buckling analysis is partly carried out at the Delft University of Technology.

A glass plate dome is analyzed with the finite element program DIANA. The study is aimed at finding the buckling behaviour of a faceted glass dome, and to provide the designer with the influence of different design parameters, like thickness of the facets and stiffness of the joints, on the stability behaviour of such a structure.

1. INTRODUCTION

The idea of the faceted glass shell structure was first put forward by Ture Wester in one of his papers on the behaviour of plate structures [1]:

“[...] if a reliable structural joint method, i.e. gluing is available, then eliminate the metal muntins as well, and leave the glass itself to carry and transfer all the forces.

Such a plate dome challenges the vision for a dome design with forces distributed so evenly, that it can be made so ‘thin’ that it becomes totally transparent and nearly disappears – a structure as clear as air, only made visible by mirroring the clouds and the sky.” [1]

The combination of readily available float glass, having high in-plane strength, with the efficiency of a plate structure therefore leads to an interesting concept of transparent construction.

Current research by Anne Bagger at the Technical University of Denmark explores the possibilities of creating such a faceted shell structure in glass. One of the crucial aspects of designing a shell structure is the buckling stability. Shell structures are highly slender structures that are loaded in compression. Consequently, they are highly sensitive to failure through buckling. This paper describes a study of the buckling stability of a glass faceted shell structure. The study is done in connection with a Master's thesis performed by the main author at the Delft University of Technology.

2. FINITE ELEMENT MODEL

In order to make the buckling analysis more transparent, a fairly straight forward design, based on the dual of a geodesic dome, is chosen for the research, see Figure 1. The span of the dome is approximately 20m. The dome is developed from the dual of a geodesic dome.

The loads that are considered in this research are an evenly distributed snow load and the self weight of the structure. In this self weight a 4mm protective glass layer is incorporated. For the calculations the design loads according to the Eurocodes are used.

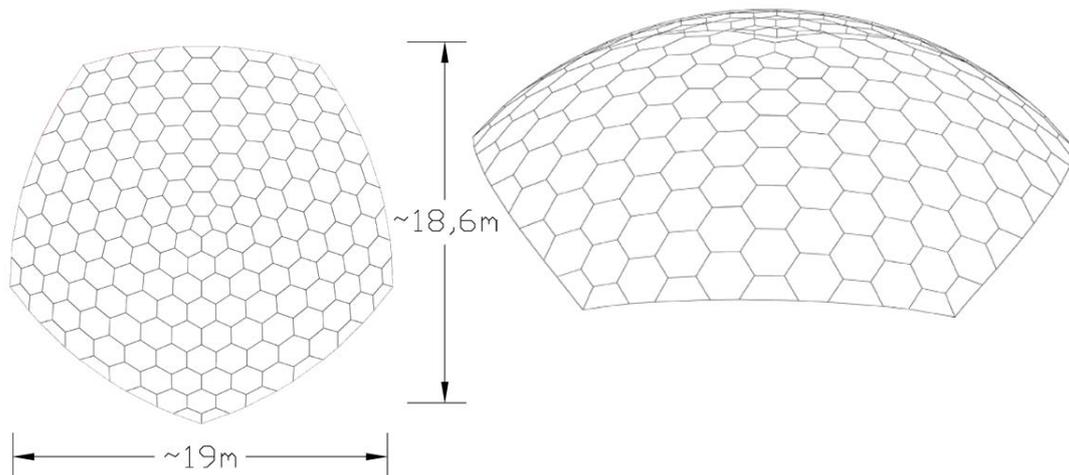


Figure 1. The design for the evaluation of the stability behaviour of the faceted glass shell structure

The dome is analysed in the finite element programme DIANA. The structural thickness of the glass facets is chosen as 16mm. The edges of the facets are connected by joints, which are modelled with the same shell elements; only a different thickness and E-modulus is used. This way the model of the joint resembles a butt-joint model. The chosen thickness is 10mm and the E-modulus is 100N/mm^2 . This corresponds to a spring stiffness against rotation of the joint of 1,67kN. By changing the thickness or E-modulus the relationship between the bending stiffness and the normal and shear stiffness of the joint can be controlled.

3. INFLUENCE OF THE PLATE BEHAVIOUR

In a buckling analysis both linear and non-linear analyses are used. Normally a linear analysis of a shell structure gives a high upper boundary for the buckling factor. This result is then normally reduced by a large 'knock-down' factor, often 6, to take imperfection sensitivity into account.

The linear buckling analysis of the faceted shell yields a low buckling factor of 2,9, see Figure 2-left. It is visible, however, that the buckling is concentrated at a single panel. A non-linear analysis of the same structure, see Figure 2-right, yields a significantly higher buckling factor of

6,64. In this buckling shape not the individual plates but the joints are actually decisive for the result.

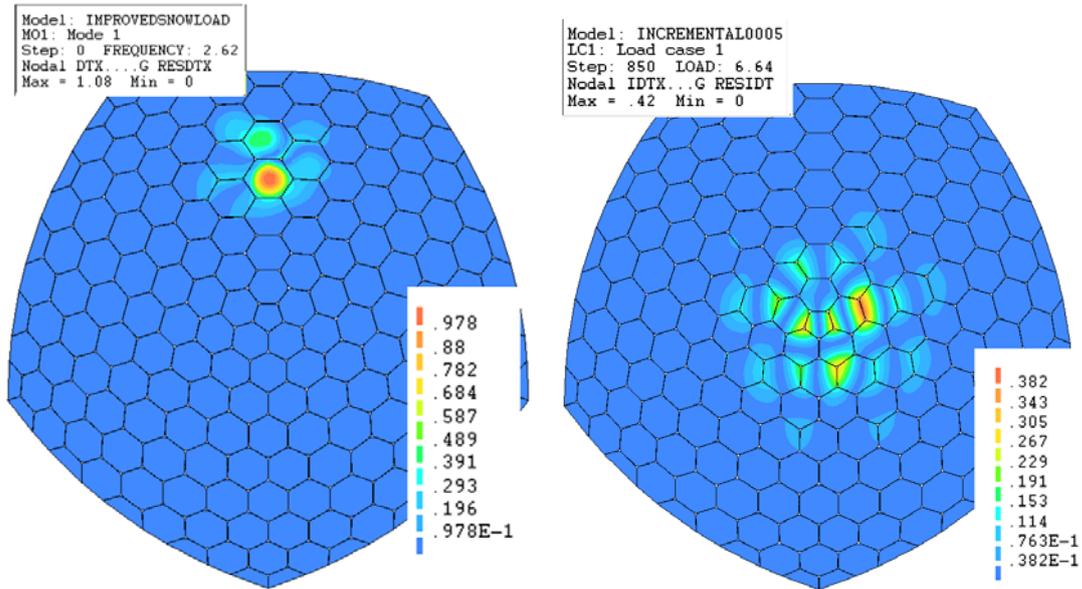


Figure 2. Left: Buckling mode 1 of the linear buckling calculation; the buckling factor $\lambda \approx 2,6$. Right: Buckling according to the non-linear buckling calculation; the buckling factor $\lambda \approx 6,6$. NB all displacements are absolute values; the right hand image therefore refers to a rippling effect. Furthermore the right hand image only shows the incremental deflection for the last load step $\Delta\lambda \approx 0,004$.

The strong difference in behaviour and result between the linear and non-linear analysis can be explained by the behaviour of the separate plates. The plates have a possibility to redistribute the loads that are exerted on them. In reality the plates start acting like hexagonal stiff frames, see Figure 3-left. Figure 3-right shows a structure built up from stiff hexagonal frames.

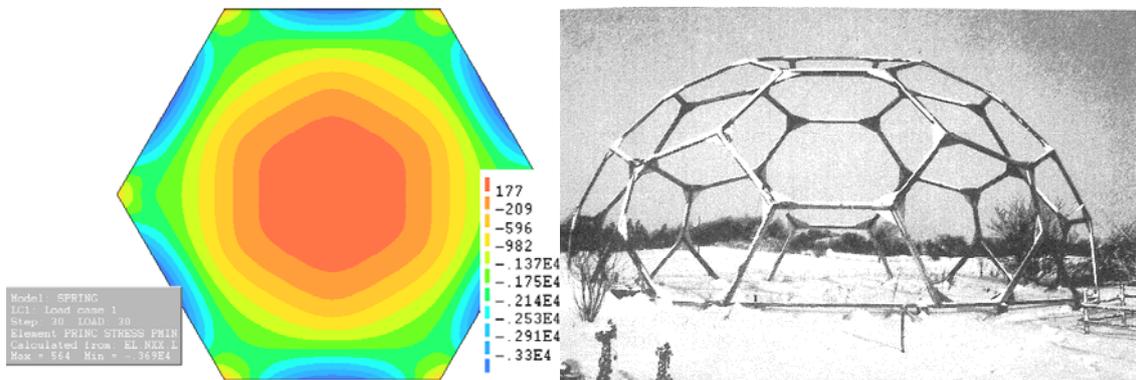


Figure 3. Left: the principle stresses show frame action developing in the plate; membrane load at $\lambda \approx 1$ is 1N/mm^2 . Right: a structure built up from stiff hexagonal frames [1]; this structure is only stable because of the frame action.

A stepped (non-linear) analysis can take this stiffening behaviour into account because it redefines the geometry of the structure at every load step. Figure 4 shows the non-linear behaviour of a single plate under different load conditions. The hexagonal plate has similar dimensions as the plates in the dome and is vertically supported along all edges and horizontally in two corners to prevent a mechanism. The theoretical (linear) buckling factor, for in-plane loads only, has a value of 19,8, with a membrane load at $\lambda \approx 1$ is 1N/mm^2 . When a non-linear calculation is made, which includes the out-of-plane load, the graph shows a kink after which the steepness of the graph slowly increases again. This is where the internal redistribution of in-

plane stresses prevents the plate from buckling. The result is that the final buckling value is almost 1,5 times the theoretical value.

The change in curvature can be made visible by strongly reducing the out-of-plane load, which is an important factor in the vertical displacement of the centre of the plate. This graph is also shown in Figure 4. The kink now clearly moved closer to the theoretical value and is much more elaborate. The redistribution of stresses again leads to a much higher buckling load.

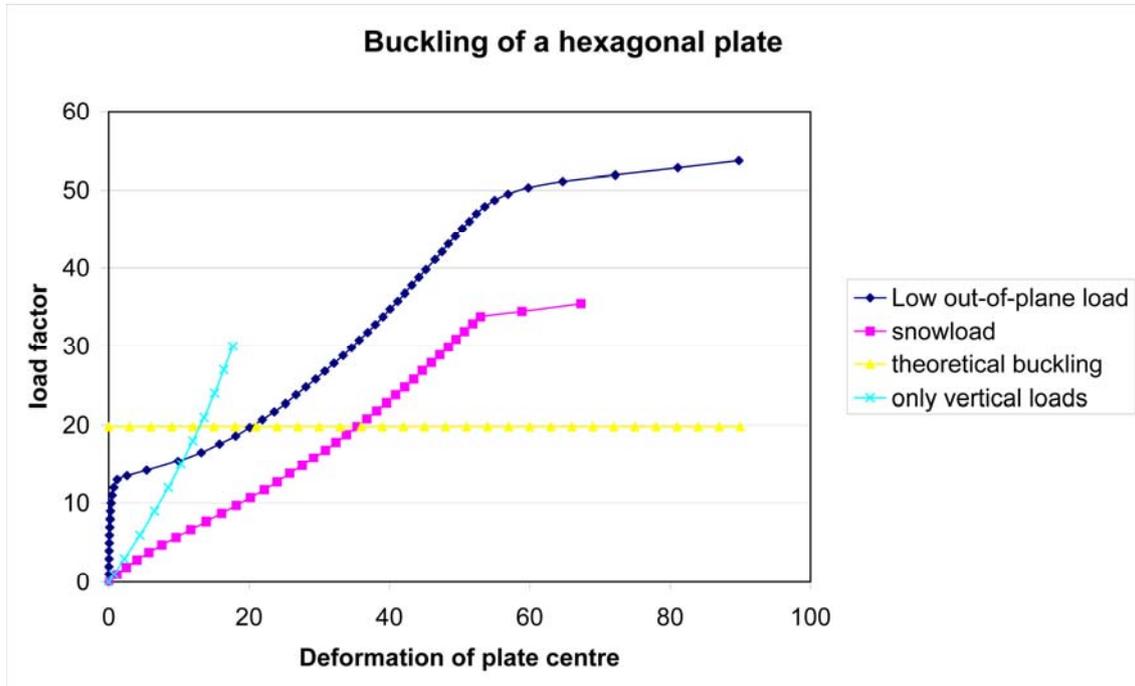


Figure 4. The buckling behaviour of a vertically supported hexagonal plate under different load conditions.

4. INFLUENCE OF THE STIFFNESS OF THE JOINT

Since the joint is very important for the behaviour of the structure, it is interesting to consider the influence of the properties of the joint. In the FEM-model the joint is a butt joint consisting of shell elements. Because the thickness and the E-modulus of these elements can be controlled the rotational spring stiffness, or bending stiffness, can be de-coupled from the normal and shear stiffness of the joint. The normal and shear stiffness, however, are both dependent on the thickness and E-modulus in the same manner and will always have a constant ratio. For simplicity the ratio between the bending and the normal stiffness will be considered in this paragraph.

First the normal stiffness (k_n) of the joint is kept constant while the bending stiffness (k_m) is varied. This is done for three different values of k_n . The unexpected result is shown in Figure 5: the bending stiffness hardly influences the overall stability of the structure. At the same time it is evident that the normal and shear stiffness apparently do influence the behaviour of the structure, since the buckling factor is significantly higher for a higher normal stiffness.

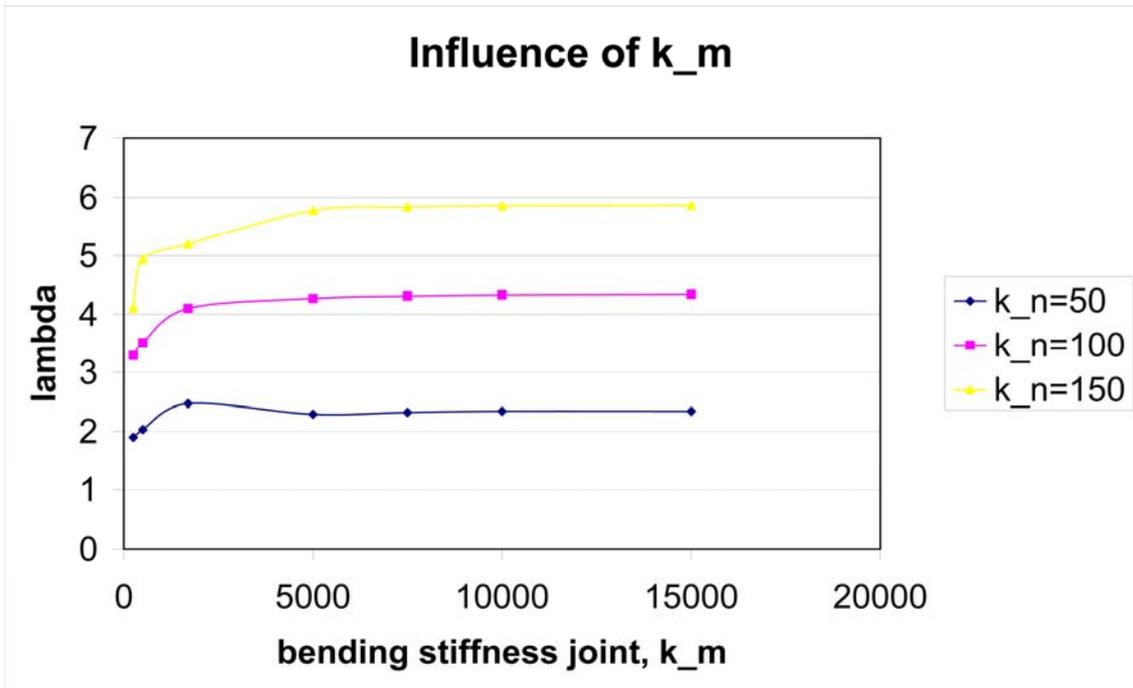


Figure 5. The influence of the bending stiffness of the joint on the buckling factor of the dome.

The direct influence of changing the normal and shear stiffness is shown in Figure 6, where the bending stiffness is kept constant at $k_m=1,67\text{kN}$. The overall stability of the structure turns out to be highly dependent on the normal and shear stiffness. This can be explained by the deformation behaviour of the joint in the model. Figure 7 shows a magnified (50x) deformation plot of the structure in which the plates attempt to slide over each other. In a 1:1 scale the plates do not actually cross over, but the magnified image shows the deformation tendency. The deformation turns out to be a combination of shear and normal deformation, see Figure 7b.

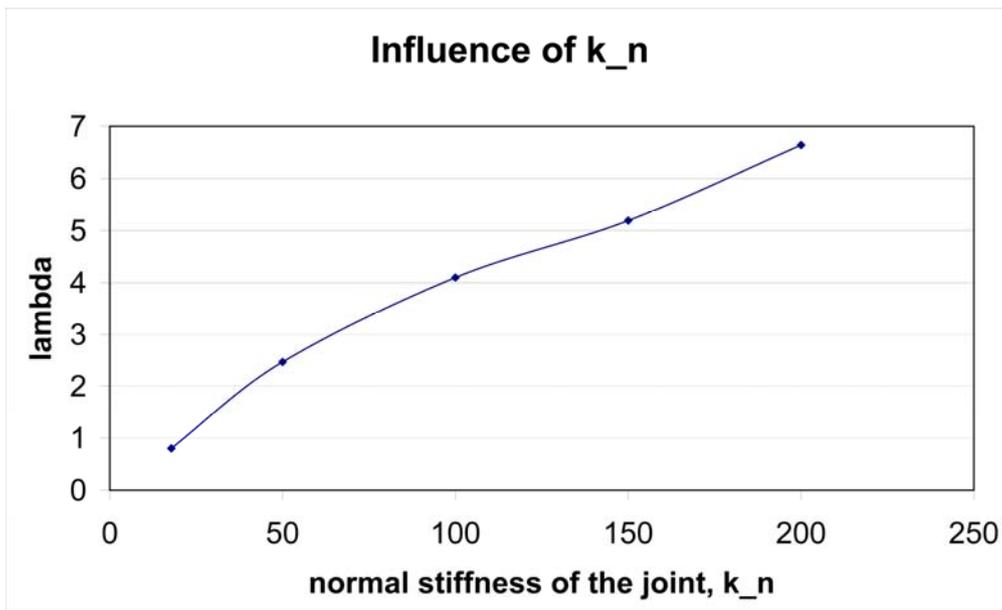


Figure 6. The influence of the normal stiffness of the joint on the buckling factor of the dome; k_m is kept constant at $k_m = 1,67\text{kN}$.

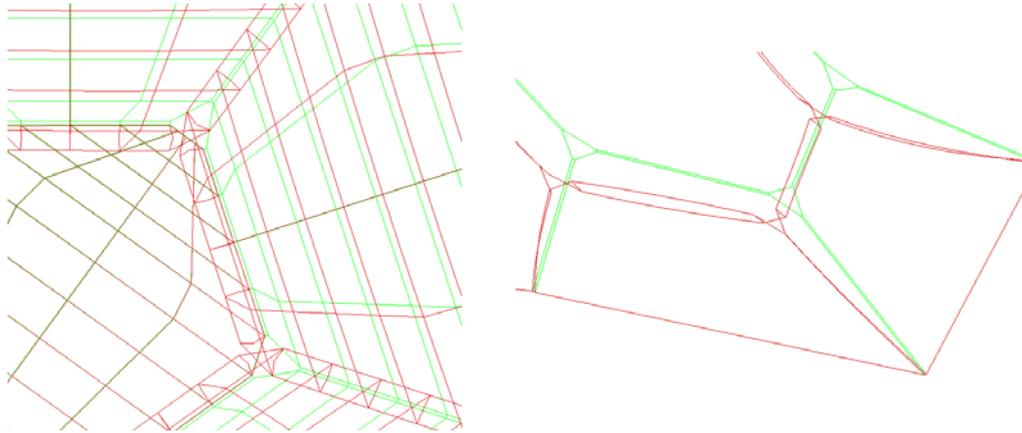


Figure 7. Deformation of the joints magnified 50x, red is the deformed shape. Note that the glass facets slide over each other. NB in scale 1:1 the glass facets do not slide over each other, the image merely shows the tendency.

The investigation shows that the design of a jointing method should not focus on the bending stiffness of the joint, but on the shear and normal stiffness. A higher bending stiffness will, however, introduce relatively larger stresses in the joint area due to the local bending of the facets themselves [3].

5. IMPERFECTION SENSITIVITY

Smooth shell structures are notorious for their sensitivity to imperfections. It can, however, be argued that the introduction of kinks (faceting) and material deviations (joints) in the faceted shell structure creates a high level of 'imperfections'. Additional imperfections might then have a relatively small influence on the lambda value. At the same time it can also be argued that dislocating a joint can have a severe impact on the structure, since the characteristics of the joint are decisive in the buckling behaviour. A dislocated joint may therefore induce buckling. This means an investigation into the imperfection sensitivity of a faceted structure is eminent.

Two important aspects need to be decided on for the investigation into the imperfection sensitivity.

1. The magnitude of the imperfection
2. The imperfection pattern

Taking into account a high degree of precision in the construction process an imperfection magnitude of 20mm is considered a realistic value.

For the shape of the imperfection three different options were chosen, dislocation of a vertex, dislocation of an entire joint and implementation of the buckling pattern of the 'perfect' structure, see Figure 2b. The last pattern forces the structure into its buckling shape before loading starts and it is usually a governing imperfection pattern. The results of the calculation are given in Figure 8.

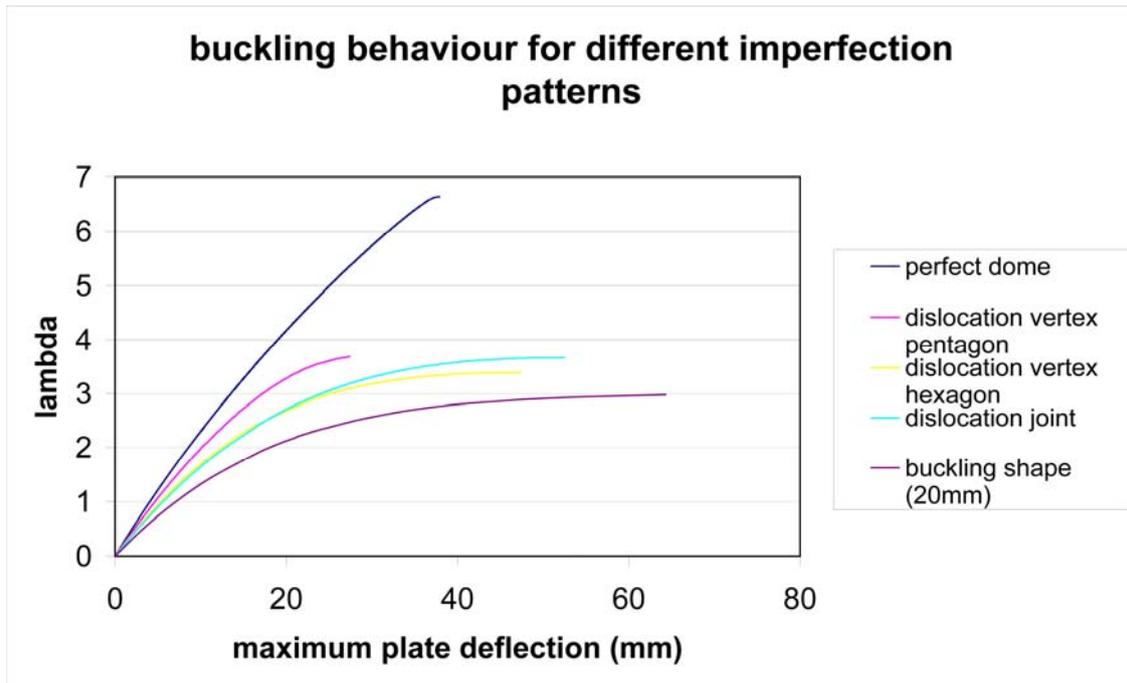


Figure 8. The influence of different imperfection patterns on the buckling factor. All dislocations are 20mm.

The introduction of the buckling patterns indeed turns out to be the governing pattern of the chosen four. The buckling factor is reduced with approximately 55% compared to the result of the perfect dome (6,6 vs. approx. 3). This means a significant reduction in the resistance of the structure. It can therefore be said that the structure is certainly sensitive to imperfections, although not as severely sensitive as smooth shells typically are.

Another interesting conclusion that can be drawn from Figure 8 is that all the imperfect structures yield buckling factors that are all in a fairly small range, between 3 and 3,4. Therefore, if an imperfection pattern exists that yields a lower buckling factor, it will be most likely also be in the same order of magnitude.

6. INFLUENCE OF THE PLATE STIFFNESS

The stiffness of the glass plates is not a fixed number. Not only the thickness of the plate can vary in design, but also the cooperation between two glass layers in a laminate are not the same for all load conditions. Under long term loads the foil interlayer will tend to creep. Especially when using a PVB foil the cooperation will need to be considered non-existing for long term loads. Therefore an investigation is made into the influence of a reduced cooperation between the glass panels.

Three scenarios have been investigated:

1. No cooperation for both self weight and snow load
2. No cooperation for self weight and full cooperation for the snow load
3. Full cooperation for both self weight and snow load.

An equivalent E-modulus is estimated for each scenario and the results are shown in Figure 9.

For a safe approximation scenario 1 is of course most suitable. However, the true behaviour of the glass laminate lies between scenario 1 and 2. A snow load is a semi-long term load and the interlayer has a lower creep when the temperature is low. Furthermore the use of a stiffer interlayer, like Glass Sentry Plus foil, will increase the cooperation between the layers of the

laminate. Figure 9 shows that the advantage that can be gained by an increased cooperation between the glass layers in the laminate is significant and therefore it will be interesting to attain a more realistic behaviour of the glass laminate.

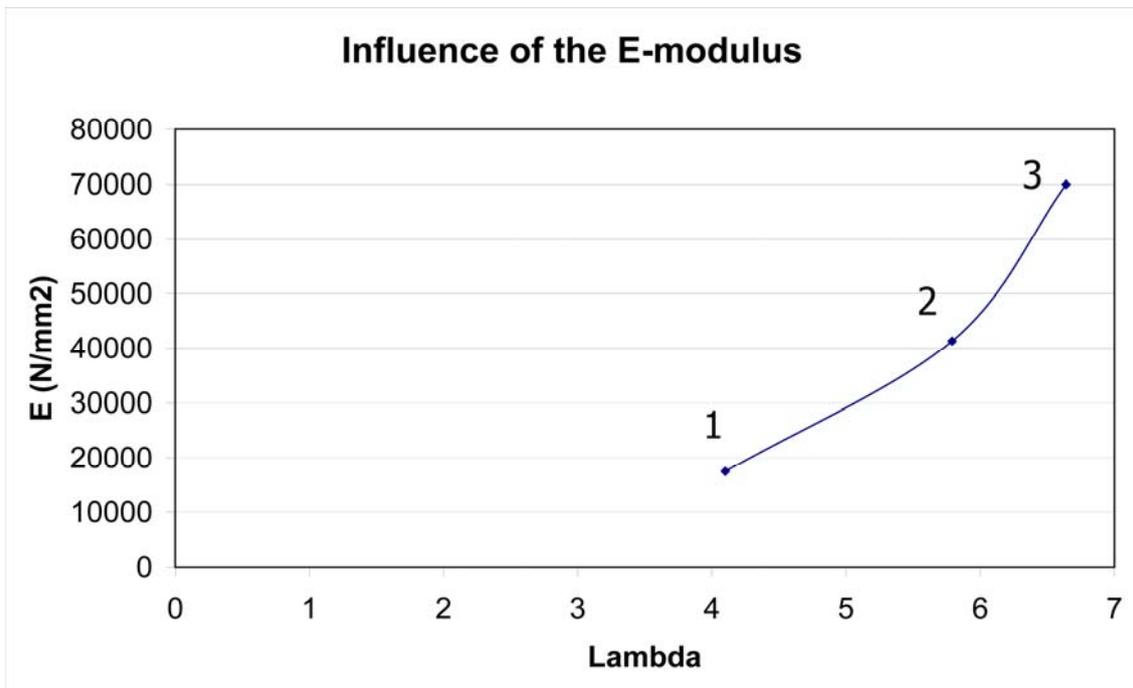


Figure 9. The influence of the stiffness of the glass facets; the numbers represent the three scenarios of cooperation. In the other investigations full cooperation was considered ($\lambda=6,6$ for the perfect geometry).

CONCLUSION

Based on of the current study it is reasonable that a faceted shell structure in glass is feasible. The concept of combining laminated float glass with a faceted structure looks very promising. Sufficiently high safety factors are found for buckling, also when realistic imperfections are introduced.

The influence of the joint stiffness on the structural behaviour is vital, and it appears that the most important task is to design a joint that has a reasonably high resistance against normal and shear deformations.

Some important aspects that still need to be investigated in relation with the stability of the structure are the introduction of asymmetric loads and the influence of different support conditions. Also parameter studies showing the dependency of shell shape and span, faceting pattern, glass thickness, etc. are necessary.

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